Sohag University
Faculty of Engineering

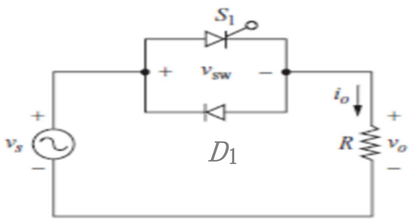
Power Electronics
Time: 2 hours

3rd Year Electrical. Dept.
Midterm exam April. 2019

Exam Model Answers

Question One (10 points)

1- A single-phase ac voltage controller Fig.1, S2 is replaced with a diode (D1) S1 operates at a delay angle α. Determine (a) an expression for rms load voltage as a function of α and Vm and (b) the range of rms voltage across a resistive load for this circuit.



Fig(1)

a)

$$V_{o,rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2(\omega t)\, d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_\alpha^{2\pi} {V_m}^2 \sin^2(\omega t) d\omega t} = \sqrt{\frac{{V_m}^2}{2\pi}\int_\alpha^{2\pi} \frac{1}{2}[1 - \cos(2\omega t)] d\omega t}$$

$$= \sqrt{\frac{{V_m}^2}{2\pi}\left[\omega t - \frac{1}{2}\sin(2\omega t)\right]_\alpha^{2\pi}} = V_m\sqrt{\frac{1}{2} - \frac{\alpha}{4\pi} + \frac{\sin(2\alpha)}{8\pi}}$$

b)

$V_{o,rms}$ max @ $\alpha = 0 \Longrightarrow = \frac{V_m}{\sqrt{2}}$

$V_{o,rms}$ min @ $\alpha = \frac{\pi}{2} \Longrightarrow =$ **$0.612V_m$**

⇨ $\mathbf{0.612V_m \le V_{o,rms} \le \frac{V_m}{\sqrt{2}}}$

2- The half-wave rectifier with a freewheeling diode has R = 2 Ω and L = 25 mH, Vm is 100 V, and the frequency is 60 Hz. Determine (*a*) the average load voltage and current, and (*b*) the power absorbed by the resistor.

a) the Fourier Series are :

$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_0 t) - \sum_{n=2,4,6...}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_0 t) \qquad (3\text{-}34)$$

The Fourier series for this half-wave rectified voltage that appears across the load is obtained from Eq. (3-34). The average load voltage is the dc term in the Fourier series:

$$V_o = \frac{V_m}{\pi} = \frac{100}{\pi} = 31.8 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{31.8}{2} = 15.9 \text{ A}$$

b)

Load power can be determined from $I_{rms}^2 R$, and rms current is determined from the Fourier components of current. The amplitudes of the ac current components are determined from phasor analysis:

$$I_n = \frac{V_n}{Z_n}$$

where $$Z_n = |R + jn\omega_0 L| = |2 + jn377(0.025)|$$

The ac voltage amplitudes are determined from Eq. (3-34), resulting in

$$V_1 = \frac{V_m}{2} = \frac{100}{2} = 50 \text{ V}$$

$$V_2 = \frac{2V_m}{(2^2 - 1)\pi} = 21.2 \text{ V}$$

$$V_4 = \frac{2V_m}{(4^2 - 1)\pi} = 4.24 \text{ V}$$

$$V_6 = \frac{2V_m}{(6^2 - 1)\pi} = 1.82 \text{ V}$$

The resulting Fourier terms are as follows:

n	V_n (V)	Z_n (Ω)	I_n (A)
0	31.8	2.00	15.9
1	50.0	9.63	5.19
2	21.2	18.96	1.12
4	4.24	37.75	0.11
6	1.82	56.58	0.03

The rms current is obtained using Eq. (2-64).

$$I_{rms} = \sqrt{\sum_{k=0}^{\infty} I_{k,rms}} \approx \sqrt{15.9^2 + \left(\frac{5.19}{\sqrt{2}}\right)^2 + \left(\frac{1.12}{\sqrt{2}}\right)^2 + \left(\frac{0.11}{\sqrt{2}}\right)^2} = 16.34 \text{ A}$$

Notice that the contribution to rms current from the harmonics decreases as n increases, and higher-order terms are not significant. Power in the resistor is $I_{rms}^2 R = (16.34)^2 2 = 534$ W.

Question Two (10points)

1- A single-phase rectifier has a resistive load of 18 Ω. Determine (*a*) the average load current, (*b*) the rms load current, (*c*) the average and rms current in each diode (*d*) the average and rms source current. (e) power factor. For a bridge rectifier with an AC source of 120 V rms and 60 Hz.

a) $V_m = 120 \times \sqrt{2} = \mathbf{169.7}$ V

$V_{o,avg} = \frac{2V_m}{\pi}$ ➔ $I_{o,avg} = \frac{2V_m}{\pi R} = \frac{2 \times 169.7}{\pi \times 18} = \mathbf{6}$ A

b) $V_{o,rms} = V_{s,rms} = \mathbf{120}$ v ➔ $I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{120}{18} = \mathbf{6.67}$ A

c) $I_{D,avg} = \frac{I_{o,avg}}{2} = \frac{6}{2} = \mathbf{3}$ A ….. $I_{D,rms} = \frac{I_{o,rms}}{\sqrt{2}} = \frac{6.67}{\sqrt{2}} = \mathbf{4.7}$ A

d) $I_{s,avg} = \mathbf{0}$ A …… $I_{s,rms} = I_{o,rms} = \mathbf{6.67}$ A

e) PF = **1** (note ➔ $I_{s,rms} = I_{o,rms}$ …., $V_{R,rms} = V_{o,rms} = V_{s,rms}$)

2- A controlled half-wave rectifier has an AC source of 240 V rms at 60 Hz. The load is a 30 Ω resistor. (*a*) Determine the delay angle such that the average load current is 2.5 A. (*b*) Determine the power absorbed by the load. (*c*) Determine the power factor.

a) $V_m = 240 \times \sqrt{2} = \mathbf{339.4}$ V

$$V_{o,avg} = \frac{1}{2\pi}\int_0^{2\pi} V(\omega t)\, d\omega t = \frac{1}{2\pi}\int_\alpha^{\pi} V_m \sin(\omega t) d\omega t = \frac{V_m}{2\pi}(1+\cos\alpha)$$

$V_{o,avg} = I_{o,avg} R = 2.5 \times 30 = \mathbf{75}$ **v**

$$\alpha = \cos^{-1}\left(\frac{2\pi V_{o,avg}}{V_m} - 1\right) = \cos^{-1}\left(\frac{2\pi \times 75}{339.4} - 1\right) = \mathbf{1.172} \text{ rad} = \mathbf{67}^\circ$$

b)

$$V_{o,rms} = \sqrt{\frac{1}{2\pi}\int_\alpha^{\pi} V^2(\omega t)\, d\omega t} = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

$$= \frac{339.4}{2}\sqrt{1 - \frac{1.172}{\pi} + \frac{\sin(2\text{x}1.172)}{2\pi}} = \mathbf{146.1 \text{ v}}$$

$I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{146}{30} = \mathbf{4.87}$ **A**

$P_R = I^2_{o,rms} R = 4.87^2 \times 30 = \mathbf{711}$ **W**

c) $PF = \frac{P}{S} = \frac{P_R}{V_{s,rms} I_{s,rms}} = \frac{711}{240 \times 4.87} = \mathbf{0.61}$

Question Third (5 points)

A controlled single-phase bridge rectifier has an AC input of 120 V rms at 60 Hz and a 20 Ω load resistor. The delay angle is 40. Determine the average current in the load, the power absorbed by the load, the source voltamperes, and the power factor.

The average output voltage is determined from Eq. (4-23).

$$V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40°\right) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current from Eq. (4-24), remembering to use α in radians.

$$I_{\text{rms}} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{\text{rms}}^2 R = (5.80)^2(20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

Question Four (5 points)

A controlled single-phase bridge rectifier has a source of 120 V rms at 60 Hz, an *RL* load where R = 10 Ω and *L* = 100 mH. The delay angle α = 60°. (*a*) Verify that the load current is continuous. (*b*) Determine the dc (average) component of the current. (*c*) Determine the power absorbed by the load.

$$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n) \qquad (4\text{-}29)$$

The dc (average) value is

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha}$$

The amplitudes of the ac terms are calculated from

$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right] \qquad (4\text{-}32)$$

$$n = 2, 4, 6, \ldots$$

(*a*) Equation (4-28) is used to verify that the current is continuous.

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75°$$

$$\alpha = 60° < 75° \qquad \therefore \text{ continuous current}$$

(*b*) The voltage across the load is expressed in terms of the Fourier series of Eq. (4-29). The dc term is computed from Eq. (4-30).

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}(120)}{\pi}\cos(60°) = 54.0 \text{ V}$$

(*c*) The amplitudes of the ac terms are computed from Eqs. (4-31) and (4-32) and are summarized in the following table where, $Z_n = |R + j\omega L|$ and $I_n = V_n/Z_n$.

n	a_n	b_n	V_n	Z_n	I_n
0 (dc)	—	—	54.0	10	5.40
2	−90	−93.5	129.8	76.0	1.71
4	46.8	−18.7	50.4	151.1	0.33
6	−3.19	32.0	32.2	226.4	0.14

The rms current is computed from Eq. (4-33).

$$I_{rms} = \sqrt{(5.40)^2 + \left(\frac{1.71}{\sqrt{2}}\right)^2 + \left(\frac{0.33}{\sqrt{2}}\right)^2 + \left(\frac{0.14}{\sqrt{2}}\right)^2 + \cdots} \approx 5.54 \text{ A}$$

Power is computed from $I_{rms}^2 R$.

$$P = (5.54)^2(10) = 307 \text{ W}$$